8.044 STATISTICAL PHYSICS I

- 1st Law
- Microcanonical ensemble
- Grand Canonical Ensemble
- Phase Transitions
- 2nd Law
- 3rd Law
- Canonical ensemble
- T=0 Quantum gases
- Transport Processes
- Probability
- Language:
PROBABILITY
Random variable (ignorance and/or QM)
Continuous, discrete, or mixed

Probability density: \( p(x) \leftrightarrow p_x(\zeta) \)

\[
\text{PROB}(\zeta \leq x < \zeta + d\zeta) = p_x(\zeta)d\zeta
\]
\[ \Rightarrow p_x(\zeta) \geq 0, \]
\[ \int_{-\infty}^{\infty} p_x(\zeta) d\zeta = 1, \]
\[ \text{PROB}(a \leq x < b) = \int_{a}^{b} p_x(\zeta) d\zeta \]

Cumulative probability:

\[ P_x(\zeta) \equiv \int_{-\infty}^{\zeta} p_x(\zeta') d\zeta' \quad \Rightarrow \quad p_x(\zeta) = \frac{d}{d\zeta} P_x(\zeta) \]

Either \( p_x(\zeta) \) or \( P_x(\zeta) \) completely specifies the RV \( x \).
Example Physical adsorption of a gas

most of the time (when hot)

small fraction of the time (when hot)

leaving the surface
\[ p(\phi) = \frac{1}{2\pi} \]

\[ 2\pi \quad \phi \]

\[ p(\theta) = 2\sin(\theta)\cos(\theta) \]

\[ \frac{\pi}{2} \quad \theta \]

\[ p(v) = \frac{v^3}{2\sigma^4} e^{-\frac{v^2}{2\sigma^2}} \]

\[ \sigma = \sqrt{\frac{kT}{m}} \]

\[ v \quad \sqrt{3}\sigma \]

\[ p(t) = \frac{1}{\tau} e^{-t/\tau} \]

\[ \tau \quad t \]
\[
\text{PROB} = p(\theta) d\theta \ p(\phi) d\phi \\
= 2 \sin(\theta) \cos(\theta) d\theta (1/2\pi) d\phi \\
d\Omega = \sin(\theta) d\theta d\phi \\
\]

\[
\text{PROB} / d\Omega = (1/\pi) \cos(\theta)
\]
Gaussian density (memorize)

\[ p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-x_0)^2}{2\sigma^2}} \]

2 parameters

\[ he^{-\frac{1}{2}} = 0.61 h \]