Example Random number generator for programmers

\[ p(x) \]

\[ p(y) \]

\[ 0 \leq x \leq 1 \]

\[ 0 \leq y \leq 1 \]

\[ x \text{ and } y \text{ are statistically independent} \]
\[ z \equiv \text{MAX}(x, y) \quad \text{Find } p(z) \]

\[ p(x, y) = p(x) p(y) \]

Where is \( \text{MAX}(x, y) = \eta \)?
Where is \( \text{MAX}(x, y) < \eta \)?

A

\[
\begin{array}{c}
0 & \eta & 1 \\
1 & \eta & p(x, y) = 1 \\
\end{array}
\]

p(x, y) = 0
**B** \[ P_z(\eta) = \eta^2 \]

**C** \[ p_z(\eta) = 2\eta \quad 0 \leq \eta \leq 1 \]

\[
\langle z \rangle = \int_0^1 2\eta^2 \, d\eta = \frac{2}{3} \left[ \eta^3 \right]_0^1 = \frac{2}{3}
\]

\[
\langle z^2 \rangle = \int_0^1 2\eta^3 \, d\eta = \frac{2}{4} \left[ \eta^4 \right]_0^1 = \frac{1}{2}
\]

\[
\text{Var}(z) = \frac{1}{2} - \frac{4}{9} = \frac{1}{18}, \quad \text{S.D.} = \frac{1}{\sqrt{18}} = 0.24
\]
Example Desorbing atom

\[ p(v, \theta, \phi) = p(v) p(\theta) p(\phi) \]

\[ p(v) = \frac{1}{2\sigma^4} v^3 \exp\left[-\frac{v^2}{2\sigma^2}\right] \]

\[ p(\theta) = 2 \sin \theta \cos \theta \]

\[ p(\phi) = \frac{1}{2\pi} \]

Find \( p(v_z) \)

leaving the surface
\[ v_z = v \cos \theta \]

\[ v \cos \theta < \gamma \]

\[ \Rightarrow v < \frac{\gamma}{\cos \theta} \]
\[ P_{vz}(\gamma) = \int_0^{\pi/2} \int_0^{\gamma / \cos \eta} p_v(\zeta) p_\theta(\eta) \, d\zeta \, d\eta \]

\[ = \int_0^{\pi/2} p_\theta(\eta) \left[ \int_0^{\gamma / \cos \eta} p_v(\zeta) \, d\zeta \right] \, d\eta \]

\[ p_{vz}(\gamma) = \frac{dP_{vz}(\gamma)}{d\gamma} = \int_0^{\pi/2} p_\theta(\eta) \left[ \frac{1}{\cos \eta} p_v(\frac{\gamma}{\cos \eta}) \right] \, d\eta \]

8.044 L6B6
\[ p_{v_z}(\gamma) = \]
\[ \int_0^{\pi/2} (2 \sin \eta \cos \eta) \left[ \frac{1}{\cos \eta} \frac{1}{2\sigma^4} \left( \frac{\gamma}{\cos \eta} \right)^3 \exp\left[-\frac{1}{2\sigma^2} \frac{\gamma^2}{\cos^2 \eta} \right] \right] d\eta \]

Let \( \frac{1}{2\sigma^2} \frac{\gamma^2}{\cos^2 \eta} \equiv X \)

\[ dX = -\frac{1}{\sigma^2} \frac{\gamma^2}{\cos^3 \eta} (-\sin \eta) d\eta \]

\[ \eta = 0 \quad \Rightarrow \quad X = \frac{\gamma^2}{2\sigma^2}; \quad \eta = \pi/2 \quad \Rightarrow \quad X = \infty \]

8.044 L6B7
\[ p_{vz}(\gamma) = \frac{\gamma}{\sigma^2} \int_{0}^{\infty} e^{-x} \, dx = -\frac{\gamma}{\sigma^2} \left[ e^{-x} \right]_{0}^{\infty} \]

\[ = \frac{\gamma}{\sigma^2} \exp\left[-\frac{\gamma^2}{2\sigma^2}\right] \quad \gamma > 0 \]