Kinematics

- Describes the geometry of motion of a particle
- Uses mathematics to describe the motion in terms of position, velocity, and acceleration
- Introduction to dynamics $\rightarrow$ study of why things move.

- Next few lectures will involve a study of translational motion.

- Simplify the physics: use concept of an ideal particle:
  - no size
  - no internal structure
  - mass
  - position as a function of time gives a complete description

- World line $\rightarrow$ position vs time

![Diagram of position and time with a worldline](image)
Speed
Consider a particle moving along a world line
- straight
- curved

Average Speed = \frac{distance travelled}{time taken} \quad \left[ \frac{L}{T} \right]

\[ s = \frac{d}{t} > 0 \quad \text{always} \quad \text{(m/s)} \quad \text{(ft/s)} \]

**Speeds**

<table>
<thead>
<tr>
<th>Light</th>
<th>(3 \times 10^8) m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sound</td>
<td>330</td>
</tr>
<tr>
<td>Man</td>
<td>12</td>
</tr>
<tr>
<td>Glacier</td>
<td>(10^{-6})</td>
</tr>
<tr>
<td>Continental Drift</td>
<td>(10^{-9})</td>
</tr>
</tbody>
</table>

Motion and speed are relative. Depend on the frame of reference where they are calculated.

Sun

[Diagram: Vector arrows indicating 12 m/s and 29.8 km/s]
Average Velocity

- 1 dimension: look at motion of a particle moving along a straight line.

- If position of particle changes with time, it is moving → velocity.

- We define an average velocity for the particle:

\[ \bar{v} = \frac{\text{change in position}}{\text{change in time}} \]

\[ \bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \]

\[ \Delta x \text{ → change in position} \]
\[ \Delta t \text{ → change in time} \]

+ sign of velocity indicates direction of motion

\[ \bar{v} = \text{slope of straight line connecting points } (x_1, t_1) \text{ and } (x_2, t_2). \]

\[ \bar{v} > 0 \text{ motion to right along } x\text{-axis} \]
\[ \bar{v} < 0 \text{ motion to left along } x\text{-axis} \]
\text{average speed} = \frac{\text{distance travelled}}{\text{time}}

\text{average velocity} = \frac{\text{change in position}}{\text{time}}

\text{Example:}

\overline{S}_{12} = \frac{2(x_3 - x_1)}{t_2 - t_1} \neq 0

\overline{v}_{12} = \frac{x_2 - x_1}{t_2 - t_1} = 0
Example.
Data for a runner

Positions and times of a runner for the initial portion of a race

<table>
<thead>
<tr>
<th>x (m)</th>
<th>t (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.31</td>
<td>0.11</td>
</tr>
<tr>
<td>0.61</td>
<td>0.18</td>
</tr>
<tr>
<td>0.91</td>
<td>0.25</td>
</tr>
<tr>
<td>1.22</td>
<td>0.31</td>
</tr>
<tr>
<td>1.52</td>
<td>0.37</td>
</tr>
<tr>
<td>1.83</td>
<td>0.43</td>
</tr>
<tr>
<td>2.13</td>
<td>0.48</td>
</tr>
<tr>
<td>2.44</td>
<td>0.54</td>
</tr>
<tr>
<td>2.74</td>
<td>0.59</td>
</tr>
<tr>
<td>3.05</td>
<td>0.64</td>
</tr>
<tr>
<td>3.66</td>
<td>0.74</td>
</tr>
<tr>
<td>4.27</td>
<td>0.84</td>
</tr>
<tr>
<td>4.88</td>
<td>0.93</td>
</tr>
<tr>
<td>5.49</td>
<td>1.03</td>
</tr>
<tr>
<td>6.10</td>
<td>1.12</td>
</tr>
<tr>
<td>6.71</td>
<td>1.20</td>
</tr>
<tr>
<td>7.32</td>
<td>1.29</td>
</tr>
<tr>
<td>7.93</td>
<td>1.37</td>
</tr>
<tr>
<td>8.53</td>
<td>1.45</td>
</tr>
<tr>
<td>9.14</td>
<td>1.53</td>
</tr>
</tbody>
</table>

- Find $\bar{v}$ for the first 1.53 seconds of race.

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{9.14 - 0}{1.53 - 0} = 5.97 \text{ m/s}.$$ 

- Find $\bar{v}$ for the time interval $t_1 = 0.54$ s and $t_2 = 0.93$.

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{4.88 - 2.44}{0.93 - 0.54} = 6.3 \text{ m/s}.$$
Example

Two automobiles make a 5h trip over a total distance of 200 km.

Average velocity for the trip:
\[ \bar{v}_A = \frac{200 \text{ km}}{5 \text{ h}} = 40 \text{ km/h} \quad \text{Car - A} \]
\[ \bar{v}_B = \frac{200 \text{ km}}{4 \text{ h}} = 50 \text{ km/h} \quad \text{Car - B} \]

Average velocity at \( t = 2h \):
\[ \bar{v}_A \approx 25 \text{ km/h} \]
\[ \bar{v}_B \approx 100 \text{ km/h} \]

B starts 1h later than A.
A + B pass each other at \( t = 2h \) and \( t = 3.5h \).
At \( t = 2.5h \), B stops for 1.5h. A speeds up.
B starts up and speeds quickly to catch A at 200 km. Both arrive at \( t = 5h \).
Instantaneous Velocity

Slope of the line \( \overline{P_1P_2} \) represents the average velocity \( \overline{v} \) between \( t_1 \) and \( t_2 \).

\[
\overline{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}
\]

Q: What is the velocity exactly at \( P_i \)?

Pick a new point closer to \( P_i \) to get a better measure.

\[
\text{Reduce } \Delta t \quad \text{reduce } \Delta x \quad \overline{v} = \frac{x_i - x_1}{t_i - t_1} = \frac{\Delta x_i}{\Delta t_i}
\]

Instantaneous velocity \( \equiv \) velocity defined as the limit as we let \( \Delta t \to 0 \).
It is equal to the slope of the tangent to the curve at the point.
\[ v(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \]

\text{\underline{calculus}}

- \textit{velocity} is the general time derivative of the position function.

\begin{center}
\begin{tikzpicture}
\draw[->] (0,0) -- (16,0) node[right] {$t$};
\draw[->] (0,0) -- (0,4) node[above] {$x$};
\draw (0,0) -- (16,0) -- (16,4) -- (0,4) -- cycle;
\draw (2,0) -- (2,4) node[above] {$200$};
\draw (4,0) -- (4,4) node[above] {$200$};
\draw (6,0) -- (6,4) node[above] {$300$};
\draw (8,0) -- (8,4) node[above] {$300$};
\draw (10,0) -- (10,4) node[above] {$400$};
\draw (12,0) -- (12,4) node[above] {$500$};
\draw (14,0) -- (14,4) node[above] {$600$};
\draw (16,0) -- (16,4) node[above] {$700$};
\draw (0,2) -- (16,2) node[below] {$0$};
\draw (0,4) -- (0,0) node[left] {$0$};
\draw (2,2) -- (2,0) node[below] {$20$};
\draw (4,2) -- (4,0) node[below] {$40$};
\draw (6,2) -- (6,0) node[below] {$60$};
\draw (8,2) -- (8,0) node[below] {$80$};
\draw (10,2) -- (10,0) node[below] {$100$};
\draw (12,2) -- (12,0) node[below] {$120$};
\draw (14,2) -- (14,0) node[below] {$140$};
\draw (16,2) -- (16,0) node[below] {$160$};
\draw (2,3) -- (2,0) node[below] {$1$};
\draw (4,3) -- (4,0) node[below] {$2$};
\draw (6,3) -- (6,0) node[below] {$3$};
\draw (8,3) -- (8,0) node[below] {$4$};
\draw (10,3) -- (10,0) node[below] {$5$};
\draw (12,3) -- (12,0) node[below] {$6$};
\draw (14,3) -- (14,0) node[below] {$7$};
\draw (16,3) -- (16,0) node[below] {$8$};
\draw (2,4) -- (2,0) node[below] {$2$};
\draw (4,4) -- (4,0) node[below] {$4$};
\draw (6,4) -- (6,0) node[below] {$6$};
\draw (8,4) -- (8,0) node[below] {$8$};
\draw (10,4) -- (10,0) node[below] {$10$};
\draw (12,4) -- (12,0) node[below] {$12$};
\draw (14,4) -- (14,0) node[below] {$14$};
\draw (16,4) -- (16,0) node[below] {$16$};
\draw (2,0) -- (2,4) node[right] {$accelerates$};
\draw (4,0) -- (4,4) node[right] {$decelerates$};
\draw (10,0) -- (10,4) node[right] {$stopped$};
\end{tikzpicture}
\end{center}

\begin{center}
\begin{tikzpicture}
\draw[->] (0,0) -- (15,0) node[right] {$t$};
\draw[->] (0,0) -- (0,3) node[above] {$v$};
\draw (0,0) -- (15,0) -- (15,3) -- (0,3) -- cycle;
\draw (2,0) -- (2,3) node[above] {$20$};
\draw (4,0) -- (4,3) node[above] {$30$};
\draw (6,0) -- (6,3) node[above] {$40$};
\draw (8,0) -- (8,3) node[above] {$50$};
\draw (10,0) -- (10,3) node[above] {$60$};
\draw (12,0) -- (12,3) node[above] {$70$};
\draw (14,0) -- (14,3) node[above] {$80$};
\draw (15,0) -- (15,3) node[above] {$90$};
\draw (0,2) -- (0,0) node[below] {$0$};
\draw (0,3) -- (0,0) node[below] {$3$};
\draw (2,2) -- (2,0) node[below] {$2$};
\draw (4,2) -- (4,0) node[below] {$4$};
\draw (6,2) -- (6,0) node[below] {$6$};
\draw (8,2) -- (8,0) node[below] {$8$};
\draw (10,2) -- (10,0) node[below] {$10$};
\draw (12,2) -- (12,0) node[below] {$12$};
\draw (14,2) -- (14,0) node[below] {$14$};
\draw (15,2) -- (15,0) node[below] {$15$};
\draw (2,3) -- (2,0) node[below] {$2$};
\draw (4,3) -- (4,0) node[below] {$4$};
\draw (6,3) -- (6,0) node[below] {$6$};
\draw (8,3) -- (8,0) node[below] {$8$};
\draw (10,3) -- (10,0) node[below] {$10$};
\draw (12,3) -- (12,0) node[below] {$12$};
\draw (14,3) -- (14,0) node[below] {$14$};
\draw (15,3) -- (15,0) node[below] {$15$};
\end{tikzpicture}
\end{center}

\textbf{Example: Racing car.}

- Position vs. time.
- Resulting velocity vs. time.
Note: Neither $\bar{v}(t)$ nor $v(t)$ depend on the choice of a coordinate system (if there is no relative motion) since they involve only differences in position.\[\leftarrow\text{invariant to choice of origins/systems}\]

Example

Ideal particle moving in a straight line with position given by:

\[x = 2.1 \, t^3 + 2.80 \, \text{(m)} \quad t \rightarrow \text{Sec.}\]

Q: What is average velocity between $t_1=3.0s$ and $t_2=5.0s$?

\[t_1 = 3.0s \quad x_1 = 2.1 \, (3.0)^3 + 2.80 = 21.7m\]
\[t_2 = 5.0s \quad x_2 = 2.1 \, (5.0)^3 + 2.80 = 55.3m\]

Average velocity $\bar{v} = \frac{x_2-x_1}{t_2-t_1} = \frac{55.3-21.7}{5.0-3.0} = 16.8 \, \text{m/s}$

Instantaneous $v$?

1. \[x = 2.1 \, t^3 + 2.80\]
2. \[x + \Delta x = 2.1 \, (t+\Delta t)^3 + 2.80\]
3. \[x + \Delta x = 2.1 \, t^3 + 4.2t(\Delta t) + 2.1(\Delta t)^2 + 2.80\]
4. \[\Delta x = 4.2t \, (\Delta t) + 2.1(\Delta t)^2\]
5. \[\bar{v} = \frac{\Delta x}{\Delta t} = 4.2t + 2.1(\Delta t)\]

Instantaneous $v(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = 4.2t$
\[ v(t=5) = 4.2 \times 5 = 21.0 \text{ m/s} \]
\[ v(t=3) = 4.2 \times 3 = 12.6 \text{ m/s} \]

Using calculus:
\[ x = 2.1t^2 + 2.80 \]
\[ v(t) = \frac{dx}{dt} = 4.2t + 0. \]
Constant Velocity Motion

- Particle moves with a position-time dependence which is a straight line.
- \( \overline{v} = \text{slope of } x(t) = \text{constant} \)

\[
\overline{v} = \frac{\Delta x}{\Delta t} = \text{constant} = v_0
\]

Also

\[
v(t) = \frac{dx}{dt} = \text{constant} = v_0
\]

\( \overline{v} = v \)
average \( \equiv \) instantaneous!!

- Motion at constant velocity is called uniform linear motion.

Let \( \overline{v} = v(t) = v_0 \), a constant.

Suppose at time \( t=0 \) the position of the particle is at \( x=x_0 \). Then at any time \( t \) its position is at \( x(t) \).

\[
\therefore \overline{v} = v_0 = \frac{x(t) - x_0}{t - 0}
\]

\[
\therefore x(t) = x_0 + v_0 t \quad \text{Eq. of a Straight Line.}
\]

\( \Rightarrow \text{General Desc. for 1-Dim. Motion at Const. } v. \)
Speed of Rifle Bullet

[15-20] min
- High speed
- Need to use special techniques

\[ s = \frac{d}{t} = \frac{\text{distance}}{\text{time}} = \text{speed} \]

Distance: 1.50 m ± 0.005 (meter-stick)

Time: 2 methods

Method I: Direct Timing

\[ \Delta t = 0.0046 \pm 0.0001 \text{ s} \]

\[ s = \frac{1.50}{0.0046} = 326 \text{ m/s} \]

Sound = 330 m/s
Method II: Rotating Shaft

- 2 paper disks on a shaft, rotating a distance d apart.
- Bullet pierces first disk
- Shaft rotates while bullet travels distance d
- Bullet pierces second disk.

1. Measure time for 1-revolution of the shaft
   \[ T_R = 0.0293 \text{s} \quad 1\text{-revolution} = 0.0293 \text{s} \]
   \[ \Rightarrow 360^\circ \]
   \[ 1800 \text{ rpm motor} \rightarrow \frac{1800}{60} = 30 \text{ rev/s} \]
   \[ \rightarrow 1 \text{ rev} = 0.0333 \text{s} \]
   (reasonable)

2. Since discs are located arbitrarily on the shaft - need to define st. line by firing bullet with shaft not rotating.

3. Mark direction of shaft rotation
4. Mark bullet reference marks
5. Rotate shaft - fire bullet
6. Measure angular displacement, \( \Delta \theta \)
Time-of-Flight

\[ \Delta t = \frac{\Delta \theta}{360^\circ} \times 0.0293 \]
\[ = \frac{77^\circ - 20^\circ}{360^\circ} \times 0.0293 \]
\[ = 0.0046 \text{ s} \]

\[ S = \frac{d}{\Delta t} = \frac{1.50 \text{ m}}{0.0046 \text{ s}} = 323 \text{ m/s} \]

Uncertainty/Errors

- Time measurement of shaft rotation
  \[ \Delta t_e = 0.0001 \]
  \[ < \frac{1}{2} \% \]

- Location of Holes (angle measurement)
  \[ \Delta \theta \sim (5 \pm 10) \%

- Length measurement
  \[ \Delta L \sim 0.01 \]