Motion with a Constant Force

\[ \vec{F} = m \vec{a} \]  
[2-nd Law]

If \( \vec{F} = \text{constant} \)
Then \( \vec{a} = \text{constant} \).

\[ \begin{align*}
\Sigma F_x &= ma_x \\
\Sigma F_y &= ma_y \\
\Sigma F_z &= ma_z
\end{align*} \] Rectangular Reference Axes.

Forces
- Tension
- Gravity
- Normal / Contact
- Friction
- Spring

Example

\[ y \text{-axis: } \quad N - mg = ma_y = 0 \]
\[ N = mg \quad \text{[No acceleration along } y \text{]} \]

\[ x \text{-axis: } \quad F = ma_x \]
\[ a_x = \frac{F}{m} \]
Problem-Solving Strategy

1. Draw a diagram indicating all key features in the problem.

2. Draw one or more free-body diagrams for the objects. For the chosen object include all the forces acting on it. Do not include any internal forces. Do not include any forces exerted by the body on some other body.

3. Select a coordinate system and show it in the free-body diagram. Determine components of the forces with reference to these axes. When the direction of acceleration is known in advance—choose that direction as +x-axis. Can choose different reference frame for each body. It must be `inertial`.

4. If there are geometrical relationships between two or more bodies—relate these algebraically!!

5. Write down Newton's Eq. of Motion for each body and solve for unknowns.

\[ F = ma \]

\[ \Sigma F_x = ma_x \]

\[ \Sigma F_y = ma_y \]

\[ \Sigma F_z = ma_z \]

6. Check special cases and extreme values of quantities. Compare with intuitive expectations. Does the result make sense?
Example

- Frictionless plane
- Massless string

Mass \( m_2 \):

\[
\begin{aligned}
F - T_2 &= m_2 a_2 \\
N_2 - m_2 q &= 0
\end{aligned}
\]  \hspace{1cm} \text{x-axis (1)}

\[
\begin{aligned}
N_2 &= m_2 q
\end{aligned}
\]  \hspace{1cm} \text{y-axis (2)}

Mass \( m_1 \):

\[
\begin{aligned}
T_1 &= m_1 a_1 \\
N_1 - m_1 q &= 0
\end{aligned}
\]  \hspace{1cm} \text{x-axis (3)}

\[
\begin{aligned}
N_1 &= m_1 q
\end{aligned}
\]  \hspace{1cm} \text{y-axis (4)}

Since bodies constrained to move together \( a_1 = a_2 = a \)

Ideal string \( T_1 = -T_2 = T \)

Add (1) + (3) \( F = (m_1 + m_2) a \)

\[
a = \frac{F}{m_1 + m_2}
\]

From (3) \( T = m_1 a = \frac{m_1}{m_1 + m_2} F \)
Ideal Pulleys/Pegs

- Are used to change the direction of force exerted by string.

- If string and pulley are both massless, the tension is the same on either side of pulley.
- If not massless then not true.

- Ideal pulley has zero mass and is also frictionless.

Normal (contact) force lies on a line which bisects the angle between the ends of the string.

Free-body diagram for an ideal (massless) string that passes over an ideal (frictionless) peg. The magnitude of the contact force is $N = 2T \cos \alpha$. 
Block + Pulley
- Frictionless surface
- Massless pulley
- Massless rope
\( m = 300 \text{ kg} \)

Pulley

\[ T - 2F = 0 \]
\[ T = 2F \]

Mass, \( m \)

\[ T = ma \]
\[ 2F = ma \]
\[ a = \frac{2F}{m} \]

\[ F = \frac{ma}{2} = \frac{300 \times 0.05}{2} = 7.5 \text{ N} \]

Pulley system provides mechanical advantage.
In this case a factor of 2.
Example

Mass $m_2$:

$$m_2 g - T = m_2 a$$  \(1\)

Mass $m_1$:

$$T = m_1 a$$  \(2\)

$$N - m_1 g = 0$$  \(3\)

Accelerations are equal since masses are connected.

Add (1) + (2)

$$m_2 g = (m_1 + m_2) a$$

$$a = \left( \frac{m_2}{m_1 + m_2} \right) g$$

Force diagram for block on a frictionless horizontal surface and for the hanging block.
Example - Inclined Plane

\[ w = mg \]

\[ mg \sin \theta = ma \]

\[ N - mg \cos \theta = 0 \]

\[ a = g \sin \theta \]

(a) A body on a frictionless inclined plane.
(b) Free-body diagram.
Ideal Strings

- Can only exert a pull (tension)
  no push (compression)

- massless: tension is the same everywhere
- mass ≠ 0: consider like any other body with mass
- string has no internal resistance and aligns itself
  with the applied force.
- assume no stretching = constant length.

Example

(a) Two blocks that are tied together with a string are pulled along by a force \( F \). The string has a length \( L \) and a mass \( m_s \). (b), (c), and (d) Free-body diagrams for the block \( m_2 \), the connecting string \( m_s \), and the block \( m_1 \), respectively.

- Tied together, all acceleration equal: \( \alpha \)

\[
\begin{align*}
\text{Mass } m_1 : & \quad F - T_1 = m_1 \alpha \quad (1) \\
\text{Mass } m_2 : & \quad T_2 = m_2 \alpha \quad (2) \\
\text{String} : & \quad T_1 - T_2 = m_s \alpha \quad (3)
\end{align*}
\]
1 + 2 + 3 \quad F = (m_1 + m_2 + m_3) a

\bar{T}_1 = F - m_1 a
= (m_2 + m_3) a

\bar{T}_2 = m_2 a
\bar{T}_1 \neq \bar{T}_2

Q: How does tension vary along string?

L: Length of string from LHS.

T: Tension at location l.

\bar{T}_1 - \bar{T}_2 = (\frac{L - l}{L}) m_3 a \quad \Box

\text{and}

\bar{T}_2 - \bar{T}_2 = \frac{l}{L} m_3 a \quad \Box

\text{Subs. for } a \text{ in } \Box

\frac{\bar{T}_2}{L} = \frac{\bar{T}_2}{L} + \frac{l}{L} m_3 \frac{F}{(m_1 + m_2 + m_3)}

\bar{T}_2 \text{ increases from } \bar{T}_2 \text{ at } l = 0 \text{ to } \bar{T}_2 = \bar{T}_1 \text{ at } l = L.

If m_3 \ll m_1 \text{ and } m_3 \ll m_2 \text{, tensions are all equal.}

\bar{T}_2 \sim \bar{T}_1 \sim \bar{T}_2
Example: Constraints

- frictionless
- massless string
- massless pulley

Mass \( m_1 \):

\[ T_1 = m_1 a_1 \]  \( \text{(1)} \)
\[ N - m_1 g = 0 \]  \( \text{(2)} \)

Mass \( m_2 \):

\[ m_2 g - T_2 = m_2 a_2 \]  \( \text{(3)} \)

Pulley:

\[ a_{T_1} - a_{T_2} = 0 \]  \( \text{(4)} \)

Constraint:

When \( m_1 \) moves a distance \( x_1 \) to the right, the mass \( m_2 \) drops a distance \( x_2 = x_1 / 2 \).

\[ \omega_2 = \frac{dx_2}{dt} = \frac{1}{2} \left( \omega_1 = \frac{dx_1}{dt} \right) \]

\[ a_2 = \frac{1}{2} a_1 \]

Solve: \[ a_2 = \left( \frac{m_2}{4m_1 + m_2} \right) g \] and \[ a_1 = 2a_2 \]
Example: Coupled Masses

Assume frictionless
massless string.

Mass $m_1$:
- Along $x$: $m_1 g \sin \theta - T = m_1 a$ \hspace{1cm} (1)
- Along $y$: $N_1 - m_1 g \cos \theta = 0$ \hspace{1cm} (2)

Mass $m_2$:
- Along $x$: $T = m_2 a$ \hspace{1cm} (3)
- Along $y$: $N_2 - m_2 g = 0$ \hspace{1cm} (4)

$1 + 3$: $m_1 g \sin \theta = (m_1 + m_2) a$

$a = \left( \frac{m_1}{m_1 + m_2} \right) g \sin \theta$
Attwood's Machine

- Frictionless, massless pulley.
- Massless rope

Mass $m_2$:
\[(m_2g - T) = m_2a\]  \(\text{(1)}\)

Mass $m_1$:
\[(T - m_1g) = m_1a\]  \(\text{(2)}\)

\(\text{From (1) + (2)}\)
\[m_2g - m_1g = \left(\frac{m_2 - m_1}{m_2 + m_1}\right) g = \frac{1}{5} g \quad (m/s^2)\]

\(\text{From (2)}\)
\[T = m_1(a + g) = m_1 \left[\frac{m_2 - m_1}{m_2 + m_1} + 1\right] g = \left(\frac{2m_1 m_2}{m_2 + m_1}\right) g\]

Pulley:
\[
\begin{align*}
T_0 &= \delta \\
(T_0 - 2T) &= 0 \\
\frac{1}{T} &= \frac{2T}{T_0} \\
T_0 &= 2T
\end{align*}
\]
Example: Accelerated Inclined Plane

- Block m on frictionless plane
- Wedge accelerated with 'a' to right

Q: What is angle θ such that block does not slip up or down the plane.

Block will not slip if its acceleration equals that of wedge.

Forces along -x \( N \sin \theta = ma \)  \( \text{(1)} \)

Forces along -y \( N \cos \theta - mg = 0 \) \( \text{(2)} \)

From \( \text{(2)} \) \( N = \frac{mg}{\cos \theta} \)

\[ \therefore \frac{mg \sin \theta}{\cos \theta} = ma \]

\[ a = g \tan \theta \]  \[ \text{[Condition on a]} \]