HW Solutions # 8 - 8.01 MIT - Prof. Kowalski

Momentum and Collisions.

1) 8.55

Please review section 8.6 rocket Propulsion in the book.

a) The average thrust is impulse divided by time:

\[ F_{\text{ave}} = \frac{J}{\Delta t} \quad (1) \]

So the ration of the average thrust to maximum thrust is:

\[ \frac{F_{\text{ave}}}{F_{\text{max}}} = \frac{J}{F_{\text{max}} \Delta \Delta t} = \frac{10}{13.3 \times 1.7} = 0.442 \quad (2) \]

b) Using the average force in equation (8.38):

\[ v_{\text{ex}} = \frac{F \Delta t}{\Delta m} = \frac{J}{\Delta m} = \frac{10}{0.0125} = 800 \text{m/s} \quad (3) \]

c) Using the result of part b in equation (8.40) - the sole equation of "rocket science":

\[ v - v_0 = v_{\text{ex}} \ln \left( \frac{m_0}{m} \right) \quad (4) \]

With \( m = m_0 - \Delta m \) and \( v_0 = 0 \) we have:

\[ v = v_{\text{ex}} \ln \left( \frac{m_0}{m_0 - \Delta m} \right) = 800 \ln \left( \frac{0.0258}{0.258 - 0.0125} \right) = 530 \text{m/s} \quad (5) \]
2) 8.73

Please refer to figure 8.41 p.322.

Using energy method including work:

\[ K_{L_1} + U_{L_1} + W_{other} = E_{L_1} + W_{other} = E_{L_2} = K_{L_2} + U_{L_2} \]  

(6)

\[ \frac{1}{2}mv_{L_1}^2 + mgy_{L_1} + W_{other} = \frac{1}{2}mv_{L_2}^2 + mgy_{L_2} \]

(7)

I will measure the gravitation potential energy with respect to the horizontal line passing the bottom of the bowl. No non-conservative force is present so \( W_{other} = 0 \).

\[ v_{L_1} = 0 \quad y_{L_1} = R \quad y_{L_2} = 0 \]  

(8)

\[ 0 + mgR = \frac{1}{2}mv_{L_2}^2 + 0 \Rightarrow v_{L_2} = \sqrt{2gR} \]  

(9)

At the bottom, due to momentum conservation law total momentum before and after sticking together is the same:

\[ \sum \vec{P} = \sum \vec{P}' \]  

(10)

Where I used "'" to denote the momentum just after the collision. The momentum at the bottom of the bowl is horizontal so we need only the component of the above vector equation in horizontal direction:

So we have

\[ mv_{L_2} + 0 = (m + m)v' = 2mv' \Rightarrow v' = \frac{v_{L_2}}{2} \]  

(11)

\( v' \) is the velocity of the total mass 2m of the two boxes. Use again the energy conservation equations (6) and (7) for "'" and "" where "" is the highest point they reach (\( v'' = 0 \)):

\[ \frac{1}{2}(m + m)v'^2 + 0 = 0 + (m + m)gy'' \Rightarrow y'' = \frac{v'^2}{2g} \]  

(12)
Combining (9) and (11) with (12):

\[
y'' = \left(\frac{\sqrt{2gR}}{4(2g)}\right)^2 = \frac{R}{4}
\]

Sensible: The height varies quadratically with the velocity, and is independent of mass. Therefore halving the velocity decreases the height by 4.

3) **8.70**

Please refer to figure 8.39 p.322.

Notations:
- Bullet mass: \( m \)
- Bullet velocity: \( v \) (is an unknown, to be found from \( \Delta x \))
- Block mass: \( M \)
- Block and bullet velocity together after the collision: \( V \)
- Compression length: \( \Delta x \)
- 0.750 N : F
- 0.250 cm: d

Writing momentum conservation

\[
\sum \vec{P} = \sum \vec{P}'
\]

in horizontal direction:

\[
mv + 0 = (m + M)V
\]

\[
v = (1 + \frac{M}{m})V
\]

The energy of the system (block and bullet) *just after* the collision is:

\[
E_1 = \frac{1}{2}(m + M)V^2
\]
The energy when at its maximum compression:

\[ E_2 = \frac{1}{2}k\Delta x^2 \]  \hspace{1cm} (16)

No nonconservative force is present after the collision so \( W_{\text{other}} = 0 \) after the collision and \( E_1 = E_2 \):

\[ \frac{1}{2}(m + M)V^2 = \frac{1}{2}k\Delta x^2 \]  \hspace{1cm} (17)

\[ V = \sqrt{\frac{k}{m + M}} \Delta x \]

From Newton’s law

\[ k = \frac{F}{d} \]  \hspace{1cm} (18)

Plugging in the numbers given in the problem:

\[ V = 2.60 \text{ m/s} \]

Using the first boxed equation and the above result you’ll get:

\[ v = 325 \text{ m/s} \]

4) 8.99

Denote the emitted neutron whose y-velocity is positive by the subscript 1 and the emitted neutron that moves in \(-y\)-direction by the subscript 2. Using conservation of momentum \( \sum \vec{P} = \sum \vec{P}' \) in the x and y directions, and neglecting the common factor of mass of a neutron,

\[ v_0 = v' \cos 10^\circ + v_1 \cos 45^\circ + v_2 \cos 30^\circ \]  \hspace{1cm} (19)

\[ 0 = v' \sin 10^\circ + v_1 \sin 45^\circ - v_2 \sin 30^\circ \]  \hspace{1cm} (20)

Where here \( v' = 2/3v_0 \).
We have 2 equations with 2 unknowns ($v_1$ and $v_2$) you can combine them to get $v_1$ and $v_2$.
Specifically you can use $\sin 45^\circ = \cos 45^\circ$, these two equations can be subtracted to eliminate $v_1$, and rearrangement gives:

\[ v_0(1 - (2/3)\cos 10^\circ + (2/3)\sin 10^\circ) = v_2(\cos 30^\circ + \sin 30^\circ) \]  

(21)

from which $v_2 = 1.01 \times 10^3 \text{ m/s}$ substitution of this into either of the momentum relations gives $v_1 = 221 \text{ m/s}$.
All that is known is that there is no z component of momentum, and so only the ratio of speeds can be determined:

\[ m_{Ba}v_{Ba} - m_{Kr}v_{Kr} = 0 \Rightarrow v_{Kr} = \frac{m_{Ba}}{m_{Kr}}v_{Ba} \]  

(22)

We don’t know what the $v_{Ba}$ is. However if we know the the released energy (it would be determined from the difference of the masses of these nuclei using the formula $E = \Delta mc^2$) you can set up the energy conservation equation. Combined with $v_{Kr} = \frac{m_{Ba}}{m_{Kr}}v_{Ba}$ we have 2 equations and two unknowns (we have already found $v_1$ and $v_2$ from momentum conservation) and in principle you can solve for $v_{Kr}$ and $v_{Ba}$.

5) 8.106

Please review section 8.6 rocket Propulsion in the book.

a) Consider system of plane + chunk of stationary air of $\Delta m$, immediately in front of the propeller. Use coordinate system in which air is initially at rest:

\[ P_1 = m_pv_p + dm(0) = m_pv_p \]  

(23)

After passing through the propeller the air chunk has:

\[ v_{air} = v_p - v_{ex} < 0 \]  

(24)
(Analogous to \(v_{\text{fuel}}\) in the book section 8.6). Here:

\[
P_2 = m_P (v_P + dv_P) + dm_{\text{air}}
\]

(25)

We ignore external force (e.g. air drag) in the system. So \(P_2 = P_1\) and hence:

\[
m_P v_P = m_P (v_P + dv_P) + dm_{\text{air}}
\]

(26)

\[
m_P \frac{dv_P}{dt} = \frac{dm_p}{dt} (v_P - v_{\text{ex}})
\]

(27)

This process accelerates the plane, and from an engineering perspective we can regard the air as generating a force on the plane:

\[
F_{\text{net}} = m_P \frac{dv_P}{dt} = (v_P - v_{\text{ex}}) \frac{dm_p}{dt}
\]

b) With the numbers given in the problem, the velocity that the propeller imparts to the air is:

\[
v_{\text{air}} = v_P - v_{\text{ex}} = \frac{F_{\text{net}}}{\frac{dm}{dt}} = \frac{1300 \text{ N}}{-150 \text{ kg/s}} = -8.66 \text{ m/s} = -31 \text{ km/h}
\]

(28)

c) Neglecting turbulence we have:

\[
P_{\text{into prop}} = P_{\text{plane}} + P_{\text{on air}} = \overrightarrow{F_{\text{net}}} \cdot \overrightarrow{v_{\text{plane}}} = \frac{1}{2} \frac{dm}{dt} (v_P - v_{\text{ex}})^2
\]

(29)

(Note that \(\frac{1}{2} \frac{dm}{dt} < 0\))

Simplify \(-\frac{1}{2} \frac{dm}{dt} (v_P - v_{\text{ex}})^2\):

\[
-\frac{1}{2} \frac{dm}{dt} (v_P - v_{\text{ex}})^2 = -\frac{1}{2} \left[ \frac{dm}{dt} (v_P - v_{\text{ex}}) \right] (v_P - v_{\text{ex}}) = -\frac{1}{2} F_{\text{net}} v_{\text{air}}
\]

(30)

(Note that \(v_{\text{air}} < 0\))
The efficiency $\epsilon$ is:

$$\epsilon = \frac{P_{\text{plane}}}{P_{\text{into prop}}} = \frac{F_{\text{net}}v_p}{F_{\text{net}}v_p - \frac{1}{2}F_{\text{net}}v_{air}} = \frac{1}{1 - \frac{v_{air}}{2v_p}} \quad (31)$$

For the numbers given in the problem:

$$\epsilon = \frac{1}{1 - \frac{-31}{2 \times 130}} = 89\% \quad (32)$$

\textbf{d)} If the diameter of the propeller were halved, the area would be $1/4$ so does the $\text{dm/dt}$ would be one fourth. We want to have the same net force and from part a you see that $v_{air} = v_p - v_{ex}$ should be multiplied by 4. This will increase the denominator, so doing this will decrease the efficiency $\epsilon$:

$$\epsilon = \frac{1}{1 - \frac{-31 \times 4}{2 \times 130}} = 68\% \quad (33)$$

It’s better to make them bigger. You can’t make it too big though because turbulence will become more and more important.