Lecture 11, Blackboard #1

Frictional Forces
- Surface in contact: 2 forces
  1. Normal: \( F_n = N \)
  2. Friction: \( f \) to surface
- Opposes relative motion or potential relative motion

Kinetic Friction
- Friction due to motion
- \( f_k = \mu_k N \)
- \( \mu_k = \text{constant} \)
- \( 0 < \mu_k < 1 \)
- \( f \) prop to \( N \)
- Opposes motion or relative motion

Example: \( \mu_k = 0.4 \), \( F_k \)?

Static Friction
- No motion
- \( \mu_s = \text{constant} \)
- \( F = \mu_s N \)
- \( F \) prop to \( N \)

Example:

- \( F = 12 \text{ N} \)
- \( f = 4 \text{ N} \)
- \( \mu_s = 0.25 \)
- \( f = 3 \text{ N} \)
- \( 36 \text{ N} \)
- \( 54 \text{ N} \)
- \( 85 \text{ N} \)

Using \( f \) equals any value needed between zero and maximum friction.

\( f \) equals frictional force for motion to start.

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Elastic Force:
- \( f = kx \)
- \( x \): displacement from rest
- \( k \): spring constant

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Frictional Force:
- \( f = \mu N \)
- \( N \): normal force
- \( \mu \): friction coefficient
- \( f \): frictional force
Example: \( \dot{a} = 0, \dot{v} = \text{constant} \)

\[ m = 100 \text{ kg}, \mu_s = 0.4, F = ? \]

Normal forces:
- \( N \sin 30^\circ - mg = 0 \)  
- \( y \)-axis: \( F \cos 30^\circ = 6 \)  
- \( f_k = \mu_k N \)  
- \( f = 6 \text{ N} \)  
- \( f_k = \mu_k N \)  
- \( F = 6 \text{ N} \)  
- \( 36 \text{ N} \)  
- \( 36 \text{ N} \)

Kinetic Friction:
- No motion
- New force to start motion
- \( f_k \leq \mu_k N \)
- \( \mu_k = \text{coefficient of kinetic friction} \)
- \( f_k \), take any value between zero and maximum value

\( f_k = \mu_k N \) when motion is about to start.
- Prop N toward force at max.
- Action vs. reaction
- Opposite lateral push trying to move body.
- Usually \( \mu_k > \mu_s \)
- Siegel on friction.

Example: Block on a Plane

\[ \begin{aligned}
\text{Example Block on Plane} &\end{aligned} \]
Lecture 11, Blackboard #3

Case 1: Statics: Just starts to slip as $\theta$ is increased.
\[
\frac{1}{\mu_{\text{max}}} = \tan \theta
\]

$\theta =$ Angle of Repose.

Case 2: Kinetic Friction
\[
\frac{f}{\mu} = \tan \theta
\]

Example

\[\begin{align*}
\text{Example: Block on a Plane} & \\
\Sigma F_x: F - f \cos 60^\circ &= ma_x \\
\Sigma F_y: N - mg \cos 60^\circ &= 0
\end{align*}\]

Assume $a_x$ up to $+F$.

\[\begin{align*}
f &= \mu \cdot N \\
f &= \mu \cdot mg \cos 60^\circ \\
N &= mg \cos 60^\circ \\
a_x &= \frac{F - mg \sin 60^\circ}{m}
\end{align*}\]

Change direction of $f$!!

\[\begin{align*}
f &= -0.55 m/s^2 \\
\text{[Block moves down plane]} & \\
F &= mg \sin 60^\circ + f = \text{max}
\end{align*}\]

Solve $a_x = -4.45 m/s^2$

Balance with assumption!!

$\mu = 0.42$

$F = 20N$

$m = 5kg$
Lecture 11, Blackboard #4

Example: Block up a Plane

\[ F = \begin{align*} F_1 &= mg \sin \theta \\ F_2 &= \text{friction} \\ F_3 &= N - mg \cos \theta \\ \end{align*} \]

\[ \Sigma F_y = 0 \]
\[ N - mg \cos \theta + f_{\text{static}} = 0 \]
\[ f_{\text{static}} = \mu N \]

\[ \Sigma F_x = 0 \]
\[ F_1 = 0 \]
\[ \text{friction} \]

\[ N = \frac{mg \cos \theta}{\mu} \]

\[ \text{Solve for } \theta \]

Ropes and Posts

\[ T = T_0 \]
\[ T_0 = \frac{N \mu}{T} \]
\[ \theta = \frac{T_0}{T} \]

Small angles

\[ \sin \theta \approx \theta \]
\[ \cos \theta \approx 1 \]
\[ \tan \theta \approx \theta \]

\[ T_0 = \frac{N \mu}{T} \]

Solve

\[ T = T_0 \]

Angle / rope ends must be equal

\[ \mu = 0.40 \]
\[ T = 2 \]
Drag Force and Terminal Speed
- Objects in fluids (air, water, etc.)
- Drag force: method of motion
- Two typical flows:
  - Laminar Flow
  - Turbulent Flow
- Drag force: velocity
- Particle size in fluid
- Terminal Velocity:
  - v_t = mg/\beta

Reactive Force on Velocity
- mg: weight (resulted force)
- buoyancy

\[ \vec{D} = -b \vec{v} \] Reaction Force
\[ \vec{v} = \text{velocity} \]
\[ \vec{F}_R = \text{drag} \]
\[ \vec{F}_D = \text{depend} 
\]
\[ \vec{F}_S = \text{strain} 
\]
\[ \vec{F}_L = \text{air} 
\]
\[ \frac{dv}{dt} = \frac{mg - bv}{m} 
\]
\[ \frac{dv}{dt} = \frac{dv}{dt} 
\]

At t increases, v increases,
and drag increases,
\[ \alpha \text{ decreases} \]
then
\[ \frac{dv}{dt} = 0 \]
\[ \text{Body move at terminal velocity } v_t \]
\[ \text{Show: } v(t) = \frac{v_0}{B} \left( \frac{1}{1 - e^{-Bt}} \right) \]
Lecture 11, Blackboard #6

Velocity, \( v = \frac{mg}{b} \)

When \( a = \frac{ds}{dt} = 0 \)

\[ \frac{dv}{dt} = \frac{b}{m} \]

\[ v(t) = \frac{mg}{b} \left( 1 - e^{-bt/m} \right) \]

Turbulent Flow

\[ D = \frac{1}{2} C_s A v^2 \]

Eff. area

S = density

C = drag coeff. (0.5 - 1.0)

Body released

\[ v(t) = \frac{mg}{b} \left( 1 - e^{-bt/m} \right) \]

When \( D = mg, a = 0 \)

\[ v(t) = v_t \text{ terminal speed} \]