18.024 Homework 5 - Solutions

Problem 1.

a) Note that $f'(a, y) = -f'(a, -y)$. So if one is positive the other one is negative, and $f'(a, y)$ can’t be positive for every $y$.

b) For a given $y$, let $f(x) = x \cdot y$. Then, $f'(x, y) = y \cdot y > 0$.

Problem 2.

a) $\nabla r = \frac{\vec{r}}{|\vec{r}|}$, so it is the unit vector in the direction of $\vec{r}$.

b) One way is by using the hint and using induction. It can also be shown straightforward:

$$\frac{\partial (r^n)}{\partial x} = \frac{\partial}{\partial x} (x^2 + y^2 + z^2)^{\frac{n}{2}} = nx(x^2 + y^2 + z^2)^{\frac{n-2}{2}} = nxr^{n-2}$$

Then,

$$\nabla (r^n) = nr^{n-2}(r)$$

c) The method used above is valid for any $n$, in particular is valid for $n = 0$ or $n$ negative.

d) It is easy to show that $f(x, y, z) = \frac{1}{2} r^2$ works.

Problem 3.

The tangent planes to spheres are perpendicular if and only if the point of intersection and the centers of the spheres form a rectangular triangle, from where the answer $c = \pm \sqrt{3}$ follows.