18.024 Homework 2 - Solutions

Let \( L = (Q, A) = \{Q + aA\} \). Note that \( M = (Q, A, P - Q) = \{Q + sA + t(P - Q)\} \) contains \( P \) (letting \( s = 0, t = 1 \)) and contains every point of \( L \) (if \( X = Q + aA, \text{let } s = a, t = 0 \)).
Now, take \( P, Q \) and \( T \in L \). They all belong to the plane that includes \( L \) and \( P \). By theorem 13.10 they define a unique plane. So \( M \) is the only plane that satisfies the conditions.

Problem 2. P. 604: 12.
\[
\begin{pmatrix}
a & b \\
c & d \\
\end{pmatrix} \cdot \begin{pmatrix}
a & b \\
c & d \\
\end{pmatrix} = \begin{pmatrix}
a^2 + bc & b(a + d) \\
c(a + d) & bc + d^2 \\
\end{pmatrix} = \begin{pmatrix}
1 & 0 \\
0 & 1 \\
\end{pmatrix}
\]
Then we have 4 equations. \( a = ±d \). If \( a = d \), we get that \( b = c = 0 \) and \( a = d = ±1 \). If \( a = -d \), \( b \) and \( c \) are arbitrary and \( a \) must satisfy \( a^2 = 1 - bc \).
Hence the solutions are: \( \begin{pmatrix}
1 & 0 \\
0 & 1 \\
\end{pmatrix} \), \( \begin{pmatrix}
-1 & 0 \\
0 & -1 \\
\end{pmatrix} \) and \( \begin{pmatrix}
a & b \\
c & -d \\
\end{pmatrix} \) with \( a^2 = 1 - bc \).

Problem 3.

Since the rows of the matrix \( A \) form a basis for \( V_n \), \( \dim(A) = n \). So the reduced echelon form will have \( n \) pivots, and the diagonal. Each of the rows will have just one non-zero entry (in the diagonal). In the reduced echelon form, the pivots are equal to 1, so the reduced echelon form of \( A \) is \( I_n \).