

Problem Set 9

Dill & Bromberg, chapter 16, problems 2, 7, 9, 12, 14, 16

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vesicle. The osmotic flow could be reversed by a sufficient concentration of a different nonexchangeable protein species B outside the membrane. The osmotic flow could be reversed because at low concentrations each distinguishable species of nonexchangeable component reduces the chemical potential independently of every other species.

16.14) Osmosis in plants. Plants must lift water from ground level to their leaves. Consider the roots as a capillary with radius 0.01cm whose walls are a semipermeable membrane with pure water on the outside and a solution inside with solute mole fraction $x = 0.001$. The solution inside has density 1 g cm^{-3} . What is the height of the solution at room temperature?

For the ideal solution given, the osmotic pressure π across a membrane with chemical potential difference $\Delta\mu$ is given by:

$\Delta\mu = \pi v$ where v is the molar volume of the solute. (equation 16.31 in text)

And, from the equation on the bottom of page 292 (derived from equation 16.39), and from the free energy of transfer equation, 16.41, we know that:

$$\Delta\mu = kTx_{\text{solute}}$$

We need it in this form because we are given $x_{\text{solute}} = 0.001$.

Remember that $\Delta\mu$ is equivalent to energy, where the total energy = kinetic energy + potential energy. (This is just one of the many equations we know for energy, but this equation is relevant here because we need to know about the downward force on the water moving up.) At equilibrium, the water is no longer rising, so there is no kinetic energy, so the $\Delta\mu$ free energy equals the gravitational potential energy at equilibrium:

$$\Delta\mu = mgh.$$

We do not have the mass, but we can get mass from the volume and the density:

$$\Delta\mu = \rho Vgh$$

The volume of the cylinder is given by: $V = \pi r^2 h$ so,

$$\Delta\mu = \rho g \pi r^2 h^2.$$

Now we know all the parameters in these two $\Delta\mu$ equations so we can set them equal to each other:

$$\rho g \pi r^2 h^2 = kTx_{\text{solute}}$$

$$h^2 = \frac{kTx_{\text{solute}}}{\rho g \pi r^2} = \frac{(4.14 \times 10^{-14} \frac{\text{g} \cdot \text{cm}^3}{\text{s}^2})(0.001)}{(1 \frac{\text{g}}{\text{cm}^3})(980 \frac{\text{cm}}{\text{s}^2})(3.14)(10^{-4} \text{cm}^2)}$$

$$h \approx 10^{-8} \text{cm}$$

Therefore, although osmotic pressure does contribute to water transport inside of trees, other factors, such as transpiration of the water from the leaves, are much more important.

16.16) Ethane association in water. For the association of ethane molecules in water, $\Delta h^0 = 2.5 \text{ kcal mol}^{-1}$ at $T = 25^\circ\text{C}$. We want to know how the association constant K changes with temperature.

From the van't Hoff equation, $\ln K = -\frac{1}{RT}[\Delta h^0 - T\Delta s^0]$, we would be able to get a value for K if we knew Δs . Though we do not know Δs , we can still get an expression for the change of K with temperature (as long as we assume that Δh^0 and Δs^0 do not depend on T).

$$\ln K = -\frac{\Delta h^0}{RT} + \frac{\Delta s^0}{R}$$

$$\frac{\partial \ln K}{\partial T} = \frac{\Delta h^0}{RT^2} = \frac{2500 \text{ cal/mol}}{(1.987 \text{ cal/molK})(298 \text{ K})^2} = 0.0142 \text{ K}^{-1}$$

As T increases, K increases, so the hydrophobic effect gets stronger.